

# 6DOF Point Cloud Alignment using Geometric Algebra-based Adaptive Filtering

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Starting with:

- 3D Point Clouds
- No initial alignment

## Standard Alignment Estimation

$$\mathcal{F}(\mathbf{R}, t) = \frac{1}{K} \sum_{n=1}^K \left\| y'_n - \mathbf{R}x'_n - t \right\|_2^2$$

$$\text{centroids: } y_n = y'_n - \bar{y} \quad x_n = x'_n - \bar{x}$$

⇒ Minimize:

$$\mathcal{F}(\mathbf{R}) = \frac{1}{K} \sum_{n=1}^K \|y_n - \mathbf{R}x_n\|_2^2$$

subject to

$$t = \bar{y} - \mathbf{R}\bar{x} \quad \mathbf{R}\mathbf{R}^T = \mathbf{R}^T\mathbf{R} = \mathbf{I}$$

**Solution: SVD**

Outlier sensitive!

Estimate correspondences:

- Keypoint extraction
- Local shape features
- We use Harris3D with SHOT [Tombari et al.]
- True and false correspondences

Recast in Geometric Algebra

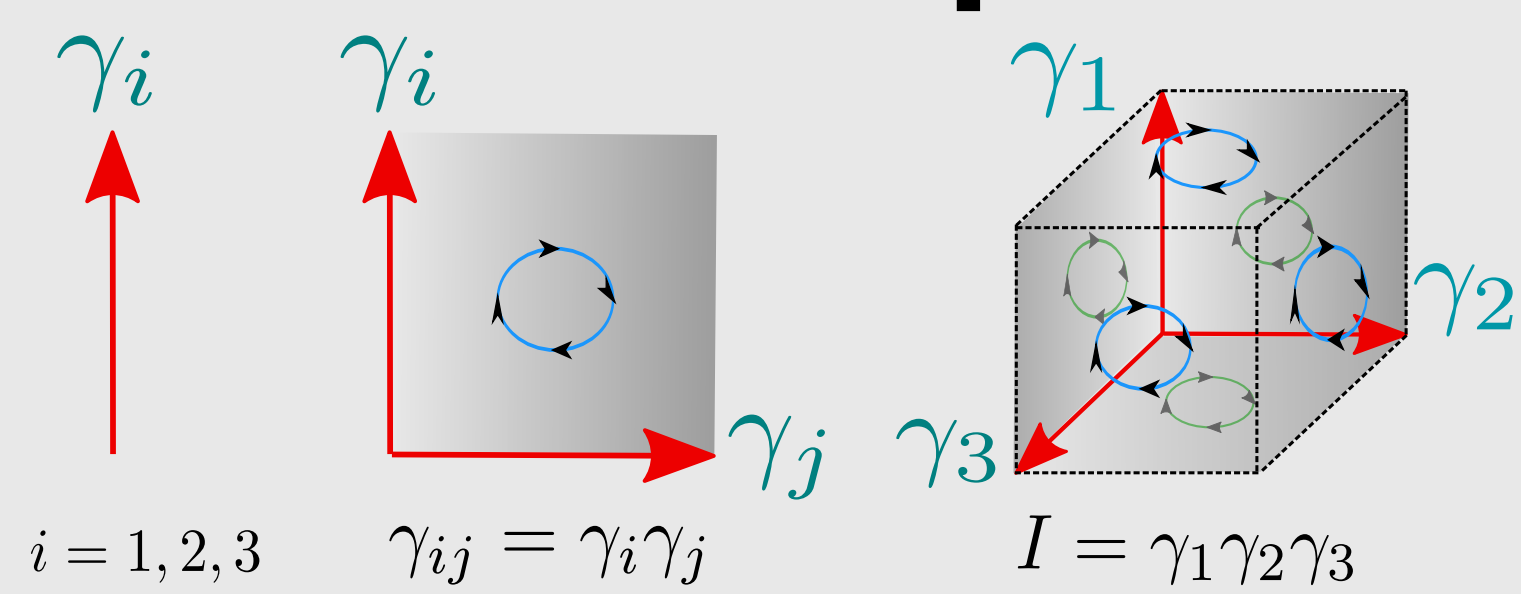
Rotation Matrix  $\mathbf{R} \rightarrow r(\cdot)\tilde{r}$  Rotors

$$J(r) = \frac{1}{K} \sum_{n=1}^K |y_n - rx_n\tilde{r}|^2$$

subject to

$$t = \bar{y} - r\bar{x}\tilde{r} \quad r\tilde{r} = \tilde{r}r = 1$$

## GA-LMS Adaptive Filter



$i = 1, 2, 3$

$\gamma_{ij} = \gamma_i \gamma_j$

$I = \gamma_1 \gamma_2 \gamma_3$

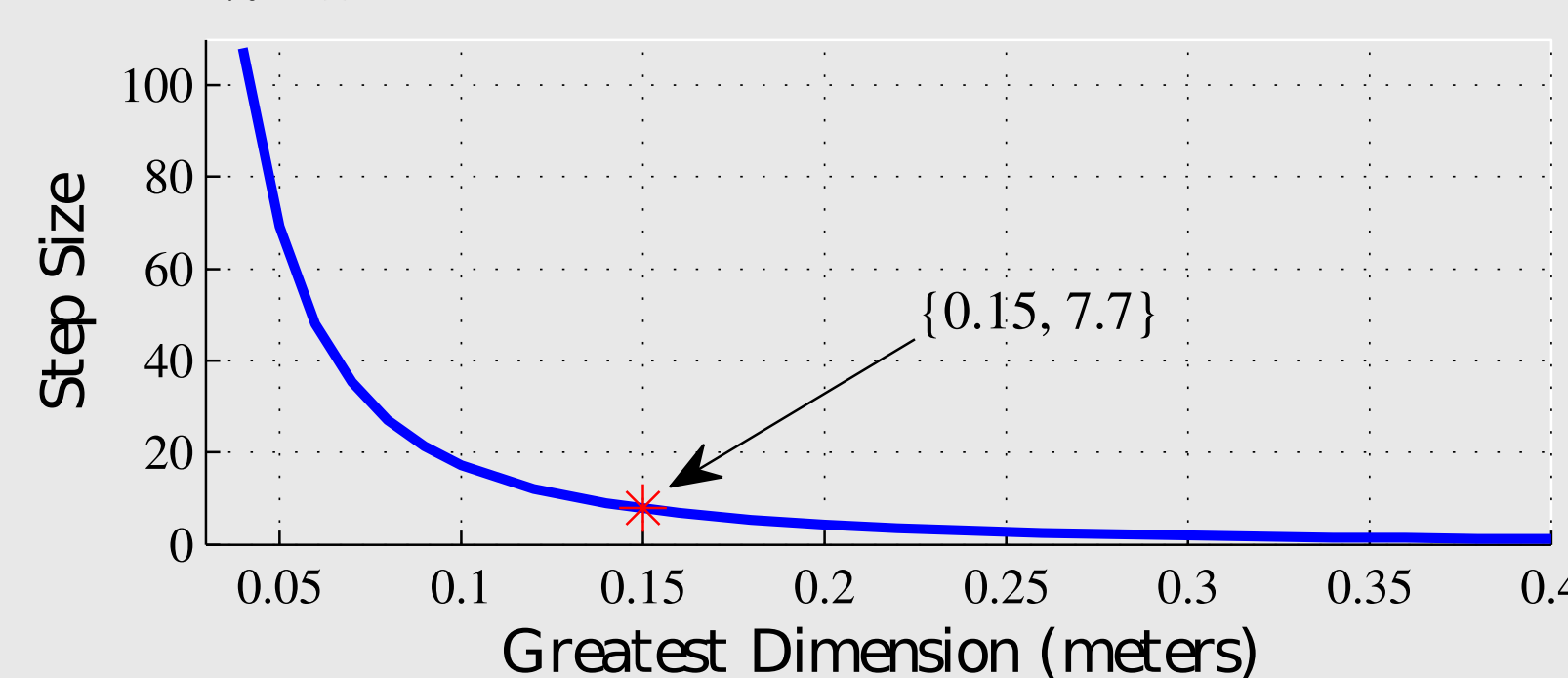
**GA-LMS Update Rule**

$$r_i = r_{i-1} + \mu [y_i \wedge (r_{i-1} x_i \tilde{r}_{i-1})] r_{i-1}$$

Selecting the step size

$$\mu(Y, X) = \rho \sum_{n=1}^K \langle y_n x_n Q \rangle / \sum_{n=1}^K \langle y_n \tilde{Q} x_n Q \rangle$$

$$Q = \sum_{n=1}^K y_n \wedge x_n \quad \rho = 15$$



**Implementation**

- GAALET - Geometric Algebra Algorithms Expression Templates

Listing 1: Implementation using Gaalet.

( $\wedge$ ) performs outer product.

( $\sim$ ) computes the reverse of a multivector.

// Update rotor

rotNew = eval(rotOld + mu \* (y ^ (rotOld \* x \* (~rotOld))) \* rotOld);

// Normalizing rotor (important!)

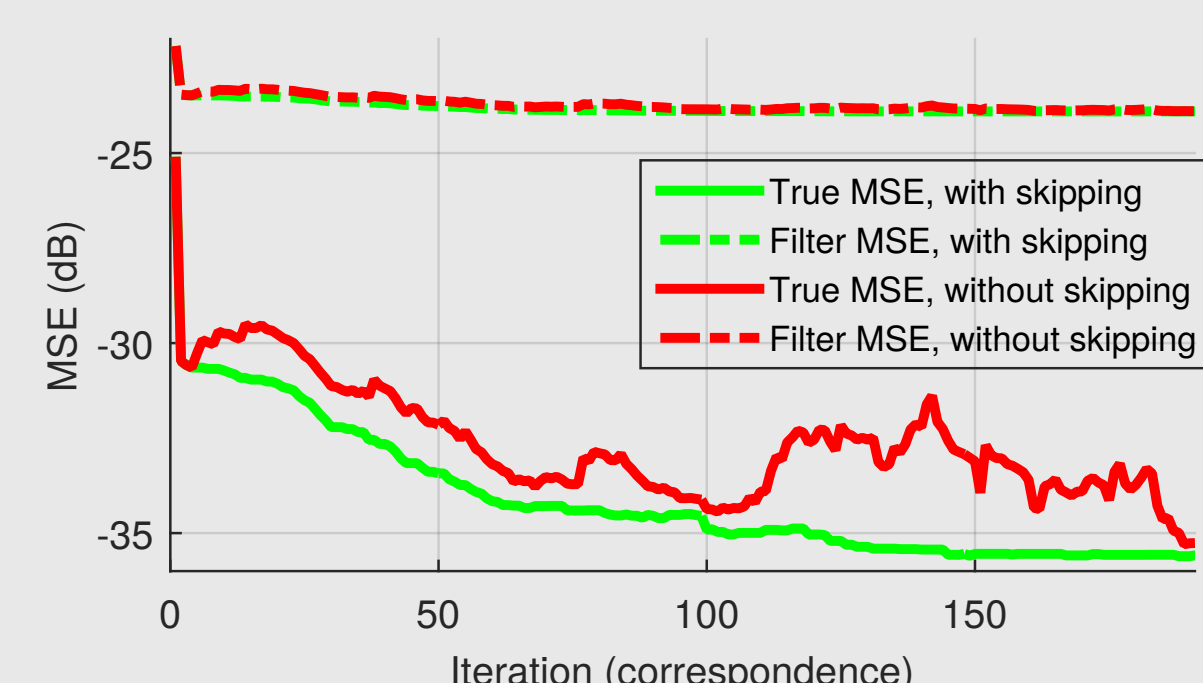
rotNew = normalize\_mv(rotNew);

The GA-LMS is an adaptive solver for the estimation phase of a 3D alignment problem

## Exploiting Adaptive Nature

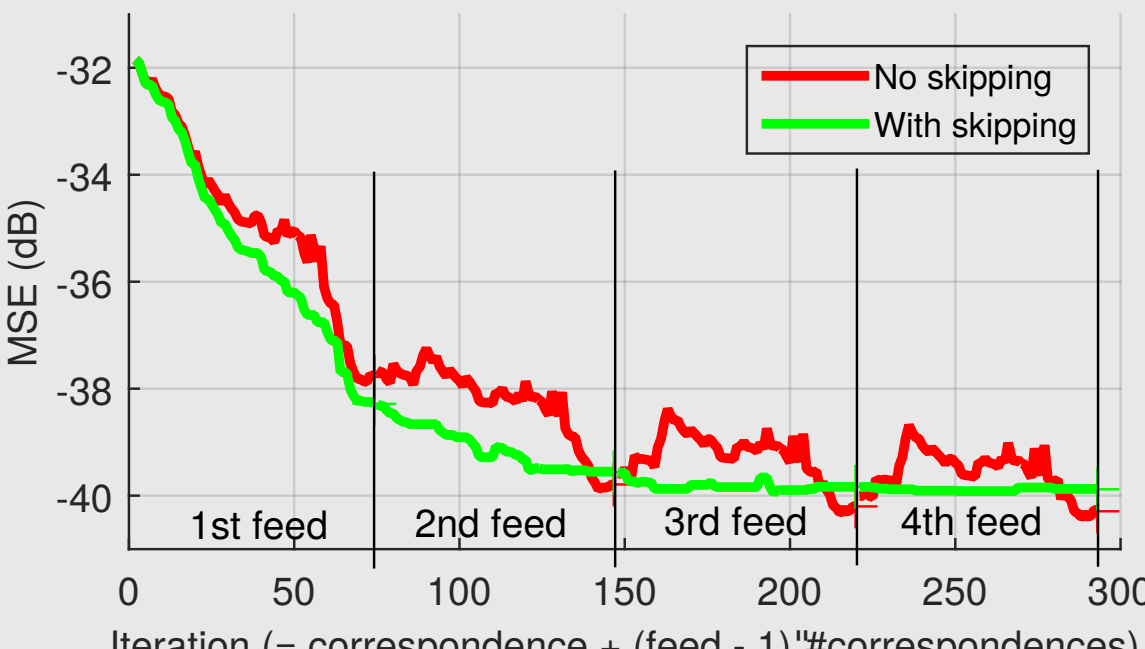
### Iteration Skipping

- Skip current MSE iteration if higher than previous
- Reduces contributions from outliers

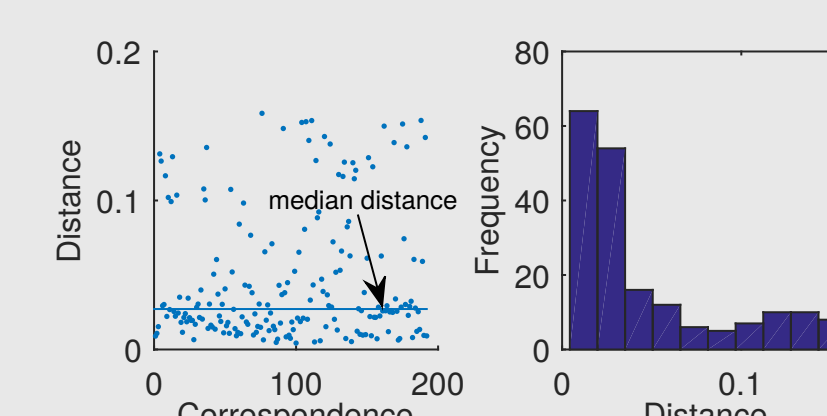


### Sample Refeeding

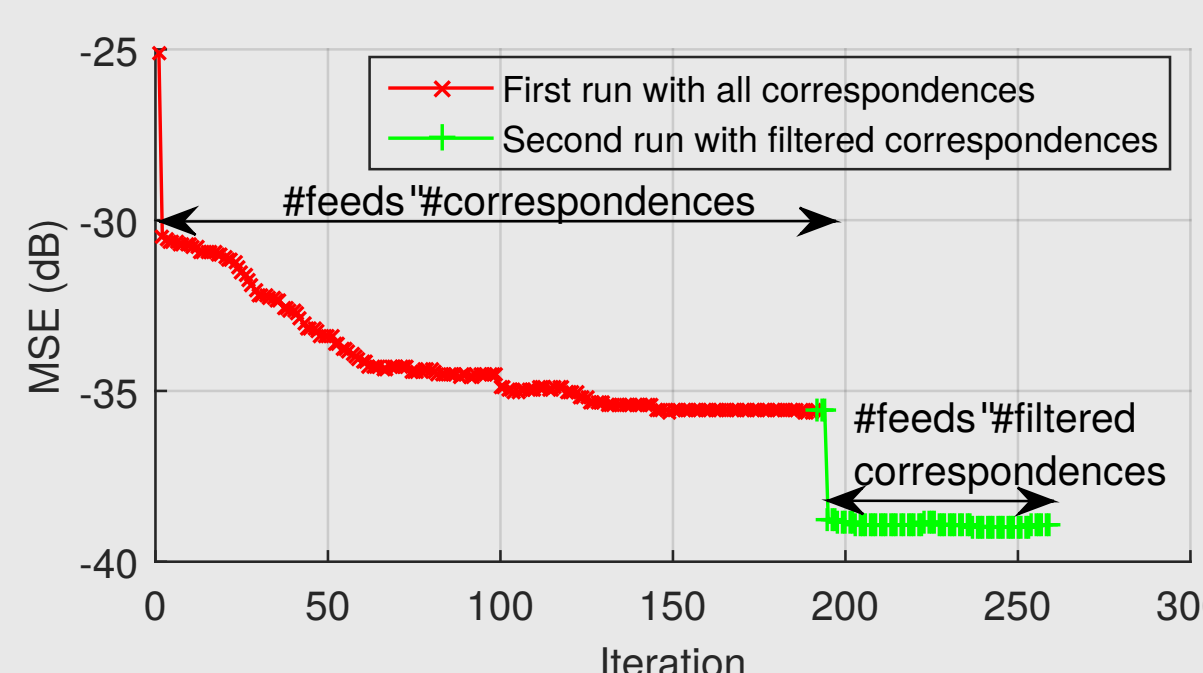
- Reprocess samples to extract more information
- Useful when number of correspondences is small and/or step size is not large enough



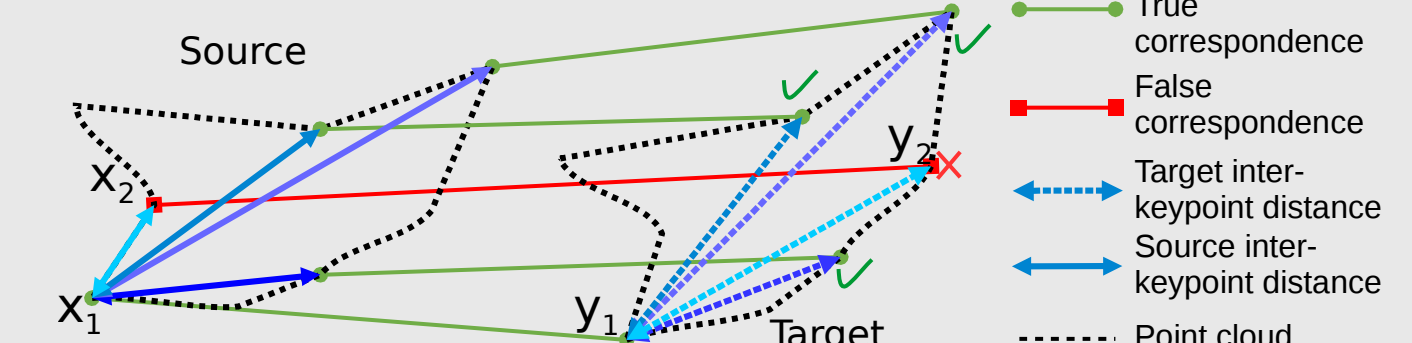
### Statistical Correspondence Filtering



- Recalculates the centroids after GA-LMS first run using median distance
- Removes 3D translation error



### Geometric Correspondence Weighting

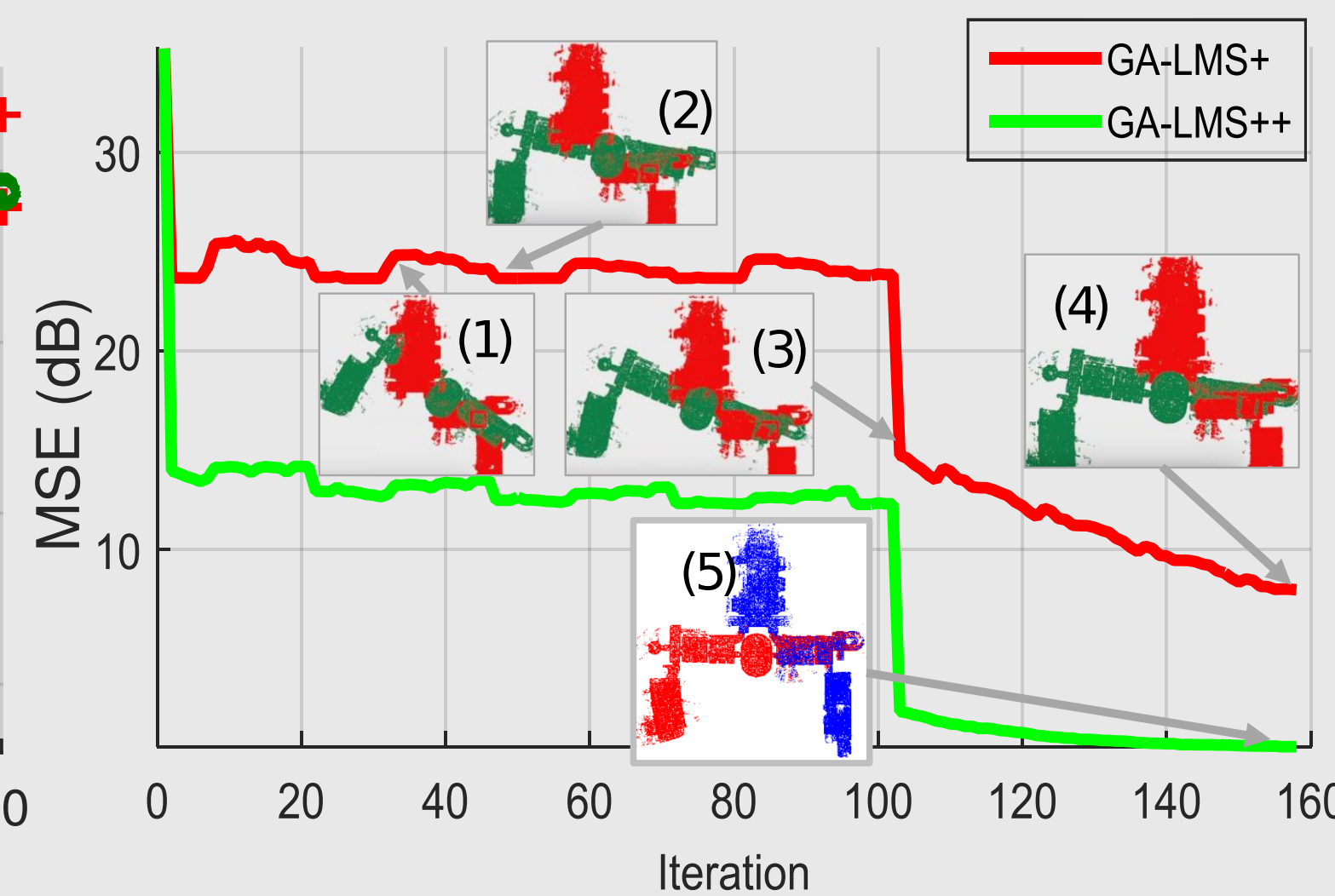
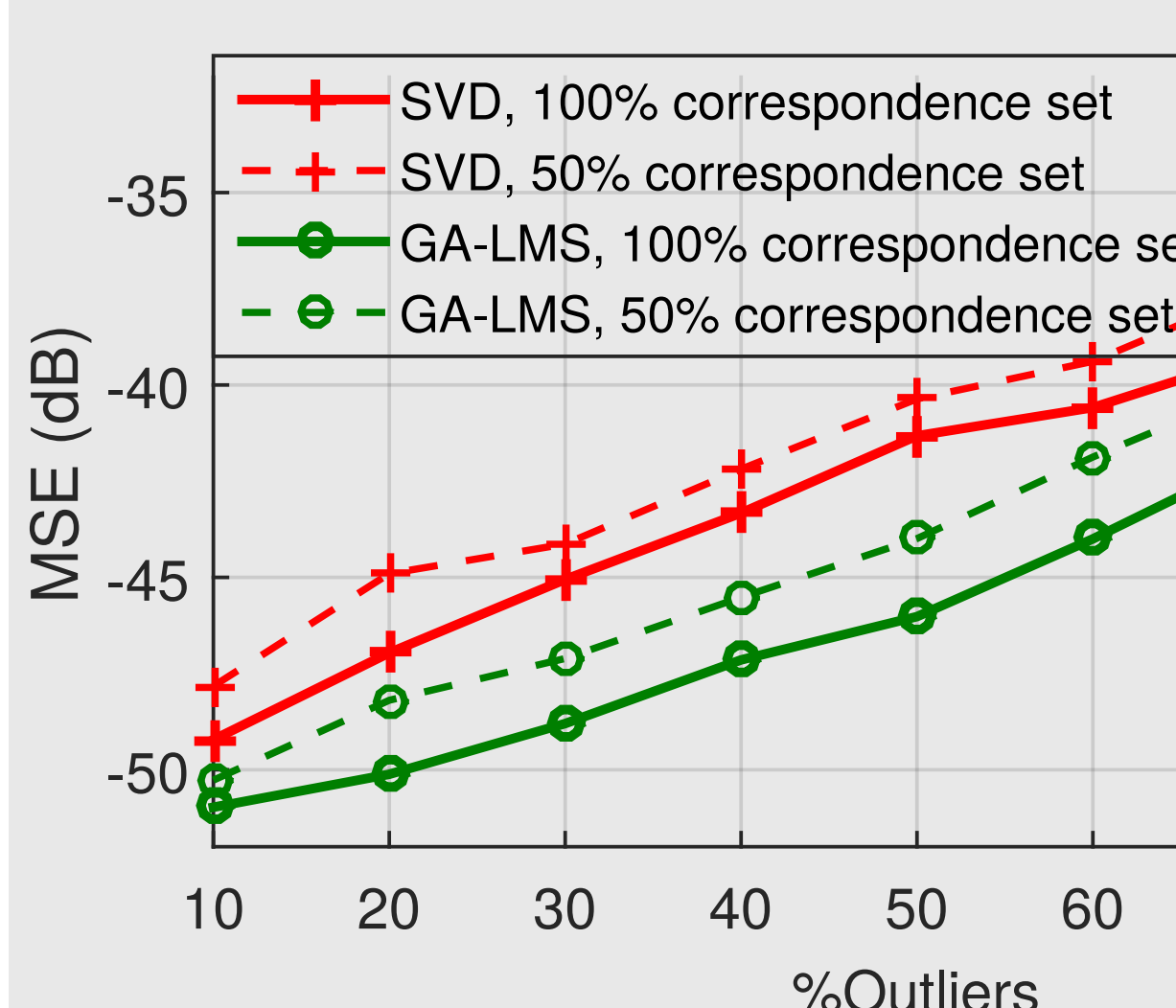
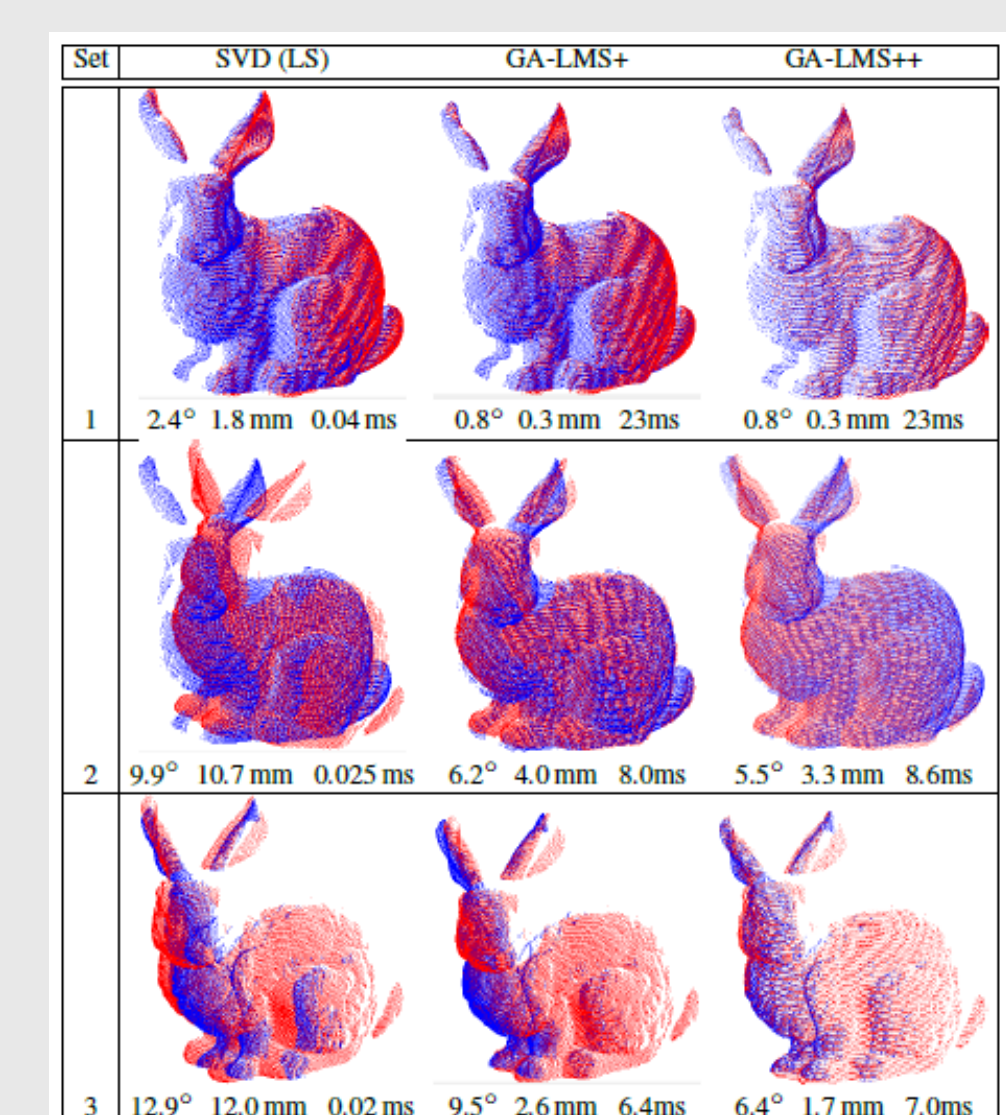


$$r_i = r_{i-1} + \alpha_i \mu [y_i \wedge (r_{i-1} x_i \tilde{r}_{i-1})] r_{i-1}$$

Filter Robustification Techniques

Test	A	B	C	D	E	F	G	H	I	J
Nr. Feeds	1	1	1	1	1	2	1	2	2	4
Skipping	x	x	x	x	x	x	x	x	x	x
Stat. filt.	x	x	x	x	x	x	x	x	x	x
Geom. w.	x	x	x	x	x	x	x	x	x	x
Angle error	16.7	14.0	10.8	8.4	24.0	14.9	15.1	6.2	7.0	5.4

## Evaluation and Comparison



- The GA-LMS is shown to be successful as the error minimizer (solver) of a 6DOF alignment algorithm
- A rule to set the GA-LMS step size as a function of the PCD dimensions and the correspondences is provided
- The adaptive nature of the GA-LMS is exploited to make it more outlier-resilient than standard SVD-based least-squares estimator
- The robustification techniques improve the performance of the original GA-LMS

See also: "Geometric-Algebra LMS Adaptive Filter and its Application to Rotation Estimation", Lopes, W. B., Al-Nuaimi, A., Lopes, C. G., at IEEE Signal Processing Letters